Breaking of Energy Conservation in Quantum Field on Subwavelength Scale

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Recently, we have predicted that the creation and destruction of energy in a wave field by classical or quantum interference break the energy conservation law in any subwavelength system. Here, we present a quantum reformulation of our model. The Hamiltonian describing the non-conservation of energy in a quantum electromagnetic field on subwavelength scale is derived.

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It is generally accepted that energy can be converted from one form to another, but it cannot be created or destroyed. According to the energy conservation law, which is the most important of conservation laws in physics, the total amount of energy in an isolated system remains constant. The conservation law affects consideration of all physical phenomena without exceptions, for an example, the energy variation in classical and quantum electromagnetic fields [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Recently, the creation and destruction of energy in a wave field on subwavelength scale, by classical or quantum interference, have been predicted [11]. Here, we present a quantum reformulation of our model. The Hamiltonian describing the non-conservation of energy in a quantum electromagnetic field on subwavelength scale is derived.

With the objective of quantizing the electromagnetic (EM) field, it is convenient to begin with the consideration of the Hamiltonian of the classical field based on Maxwell's equations [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Let us first consider a single-mode EM field in free space, which is a superposition of N time-harmonic plane waves having different phases. The vector potential of the n-th linearly polarized wave is assumed to be $\mathbf{A}_n(\mathbf{r},t) = \mathbf{a_k}e^{i\mathbf{kr}+i\varphi_n} + \mathbf{a^*_k}e^{-i\mathbf{kr}-i\varphi_n}$, where $\mathbf{k} \equiv (k_x, k_y, k_z)$ is the wave vector and φ_n is the wave phase. The vector potential $\mathbf{A}(\mathbf{r},t) = \sum_{n=1}^{N}\mathbf{A}_n(\mathbf{r},t)$ determines the electric $\mathbf{E}(\mathbf{r},t) = -\frac{1}{c}\dot{\mathbf{A}}$ and magnetic $\mathbf{H}(\mathbf{r},t) = \nabla\times\mathbf{A}$ components of the field. One can easily find the Hamiltonian $\mathcal{H}$ of the field by calculating the field energy $\mathcal{E} = \frac{1}{8\pi}\int(\mathbf{E}^2+\mathbf{H}^2)dV$:

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}(\mathbf{P}_n\mathbf{P}_m + \omega^2\mathbf{Q}_n\mathbf{Q}_m) = \tag{1}$$

$$= \sum_{n=1}^{N}\mathcal{H}_{nn} + \sum_{n\neq m}^{N}\sum_{m\neq n}^{N}\mathcal{H}_{nm}, \tag{2}$$

where

$$\mathbf{Q}_n = \left(\frac{V}{4\pi c^2}\right)^{1/2}(\mathbf{a_k}e^{i\mathbf{kr}+i\varphi_n} + \mathbf{a^*_k}e^{-i\mathbf{kr}-i\varphi_n}) \tag{3}$$

and

$$\mathbf{P}_n = -i\omega\left(\frac{V}{4\pi c^2}\right)^{1/2}(\mathbf{a_k}e^{i\mathbf{kr}+i\varphi_n} - \mathbf{a^*_k}e^{-i\mathbf{kr}-i\varphi_n}) \tag{4}$$

are the canonical variables, and V is the volume. To calculate the Hamiltonian of spherical waves, in the potential $\mathbf{A}_n(\mathbf{r},t)$, the term $e^{\pm i\mathbf{kr}\pm i\varphi_n}$ should be replaced by $r^{-1}e^{\pm i\mathbf{kr}\pm i\varphi_n}$. The first term in the expression (2) is the traditional Hamiltonian of a classical single-mode EM field [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The Hamiltonian does not take into account the interference (correlation) between the waves. The total energy of an ensemble of the non-correlated (non-coherent) waves, which are described by this Hamiltonian, is always conserved: $\langle\mathcal{H}\rangle = \sum_{n=1}^{N}\langle\mathcal{H}\rangle_{nn} = N\mathcal{E}_1$. The positive or negative cross-correlation energy (second term) is responsible for the energy non-conservation associated with the phase correlation between the waves. The energy can be created or completely destroyed in an ensemble of coherent waves ($0 \leq \langle\mathcal{H}\rangle \leq N^2\mathcal{E}_1$) by modification of the wave phases φ_n. At appropriate conditions, the phase modification may require an amount of energy that is negligible compared to the energy of waves. In such a case, the interference ("mixing") of waves completely destroys the energy if the waves interfere destructively in all points of a physical system. The interference creates the energy if waves interfere only constructively.

The usual form of the Hamiltonian for the quantum field $\hat{\mathbf{A}}$ is

$$\hat{\mathcal{H}} = \frac{1}{8\pi}\int(\hat{\mathbf{E}}^2 + \hat{\mathbf{H}}^2)dV. \tag{5}$$

The quantum form of the Hamiltonian (1) can be found by using the standard procedure [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] based on the replacement of the canonical variables (3) and (4) by the operators $\hat{\mathbf{Q}}_n$ and $\hat{\mathbf{P}}_n$:

$$\hat{\mathcal{H}} = \frac{\hbar\omega}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}(\hat{\mathbf{a}}^{\dagger}_{\mathbf{k}}e^{-i\varphi_m}\hat{\mathbf{a}}_{\mathbf{k}}e^{i\varphi_n} + \hat{\mathbf{a}}_{\mathbf{k}}e^{i\varphi_m}\hat{\mathbf{a}}^{\dagger}_{\mathbf{k}}e^{-i\varphi_n}), \tag{6}$$

where $\hat{\mathbf{a}}^{\dagger}_{\mathbf{k}}$ and $\hat{\mathbf{a}}_{\mathbf{k}}$ are Dirac creation and destruction operators, respectively. Here, we used the non-conventional commutation relations to take into account the interference correlation between the waves: $[\hat{\mathbf{a}}_{\mathbf{k}n}, \hat{\mathbf{a}}^{\dagger}_{\mathbf{k}m}]=1$ and $[\hat{\mathbf{a}}_{\mathbf{k}n}, \hat{\mathbf{a}}_{\mathbf{k}m}]=[\hat{\mathbf{a}}^{\dagger}_{\mathbf{k}n}, \hat{\mathbf{a}}^{\dagger}_{\mathbf{k}m}]=0$.

The Hamiltonian can be written also in the more convenient form:

$$\hat{\mathcal{H}} = \sum_{n=1}^{N} \hat{\mathcal{H}}_{nn} + \sum_{n\neq m}^{N} \sum_{m\neq n}^{N} \hat{\mathcal{H}}_{nm}, \tag{7}$$

where

$$\hat{\mathcal{H}}_{nn} = \hbar\omega \left(\hat{\mathcal{N}}_{\mathbf{k}} + \frac{1}{2} \right) \tag{8}$$

and

$$\hat{\mathcal{H}}_{nm} = \frac{\hbar\omega}{2} [\hat{\mathcal{N}}_{\mathbf{k}} e^{-i\varphi_m + i\varphi_n} + +(\hat{\mathcal{N}}_{\mathbf{k}} + 1) e^{i\varphi_m - i\varphi_n}]. \tag{9}$$

Here, $\hat{\mathcal{N}}_{\mathbf{k}} = \hat{\mathbf{a}}^{\dagger}_{\mathbf{k}} \hat{\mathbf{a}}_{\mathbf{k}}$ is the photon number operator. Notice that the vacuum-energy part in Eq. (9) is a real value under the commutation relations: $[\hat{\mathbf{a}}_{\mathbf{k}n}, \hat{\mathbf{a}}^{\dagger}_{\mathbf{k}m}] = \pm e^{-i\varphi_m + i\varphi_n}$ and $[\hat{\mathbf{a}}_{\mathbf{k}n}, \hat{\mathbf{a}}_{\mathbf{k}m}]$=$[\hat{\mathbf{a}}^{\dagger}_{\mathbf{k}n}, \hat{\mathbf{a}}^{\dagger}_{\mathbf{k}m}]$=0.

The quantum electric and magnetic fields are determined by the quantized vector potential that has the form:

$$\hat{\mathbf{A}}_n = \hat{\mathbf{a}}_{\mathbf{k}} \mathbf{A}_{\mathbf{k}} + \hat{\mathbf{a}}^{\dagger}_{\mathbf{k}} \mathbf{A}_{\mathbf{k}}{}^{*}_{\mathbf{k}}, \tag{10}$$

where

$$\mathbf{A}_{\mathbf{k}} = \left(\frac{2\pi c^2}{\hbar\omega V} \right)^{1/2} e^{i\hbar\mathbf{k}\mathbf{r} + i\varphi_n}. \tag{11}$$

The first term in Eq. 7 is the traditional Hamiltonian of a quantum single-mode EM field [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The Hamiltonian does not take into account the quantum interference between the different waves. The total energy of an ensemble of the N waves describing by the traditional Hamiltonian is $\langle\mathcal{E}\rangle = N\hbar\omega(\langle\mathcal{N}_{\mathbf{k}}\rangle + \frac{1}{2})$. The field corresponding to this Hamiltonian is usually considered as a linear superposition of $N\langle\mathcal{N}_{\mathbf{k}}\rangle$ particles, Einstein's photons having the energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$. The wave function of the field is a linear superposition of Fock's (number) noncorrelated states $|n\rangle$. According to our model, however, the field should be considered as a linear superposition of N correlated (entangled) states each of which has the energy $\langle\mathcal{N}_{\mathbf{k}}\rangle\hbar\omega$ and momentum $\langle\mathcal{N}_{\mathbf{k}}\rangle\hbar\mathbf{k}$. In such a representation, the each state of N correlated waves contains $\langle\mathcal{N}_{\mathbf{k}}\rangle$ entangled photon states $|n\rangle$. The two representations are equivalent in the field model describing by the traditional Hamiltonian. In the frame of such a model, the energy $\langle\mathcal{E}\rangle = N\hbar\omega(\langle\mathcal{N}_{\mathbf{k}}\rangle + \frac{1}{2})$ and the total number $N\langle\mathcal{N}_{\mathbf{k}}\rangle$ of photons are conserved under interfering ("mixing") the quantum states. There are always nonzero fluctuations of energy about its zero ensemble average even for a vacuum state, $\langle\mathcal{E}\rangle = N\hbar\omega/2$.

The situation is completely different in the case of the field describing by the full Hamiltonian (7). The second term in the expression (7), which is responsible for the energy non-conservation associated with the phase correlations between the waves, provides the positive or negative cross-correlation energy. The energy can be created or completely destroyed by interfering ("mixing") the N quantum waves, $0 \leq \langle\mathcal{E}\rangle \leq N^2\hbar\omega(\langle\mathcal{N}_{\mathbf{k}}\rangle + \frac{1}{2})$. The respective number of photons varies from zero to $N^2\langle\mathcal{N}_{\mathbf{k}}\rangle$. At the appropriate phase conditions the zero fluctuations in the field energy can be obtained in both the vacuum and non-vacuum states. According to the Hamiltonian (7), the total energy of an ensemble of N non-correlated waves containing $\langle\mathcal{N}_{\mathbf{k}}\rangle$ non-correlated photons is always conserved, $\langle\mathcal{E}\rangle = N\hbar\omega(\langle\mathcal{N}_{\mathbf{k}}\rangle + \frac{1}{2})$. The total number of non-correlated photons $N\langle\mathcal{N}_{\mathbf{k}}\rangle$ is constant.

So far we have considered a superposition of the single-mode waves. To find the energy of a multimode classical or quantum field one should use the above model for the multimode vector potential having the form $\mathbf{A}(\mathbf{r},t) = \sum_{\mathbf{k}} \mathbf{A}_{\mathbf{k}}(\mathbf{r},t)$, where $\mathbf{A}_{\mathbf{k}}(\mathbf{r},t) = \sum_{n=1}^{N_{\mathbf{k}}} \mathbf{a}_{\mathbf{k}} e^{i\mathbf{k}\mathbf{r} + i\varphi_{n\mathbf{k}}} + \mathbf{a}^{*}{}_{\mathbf{k}} e^{-i\mathbf{k}\mathbf{r} - i\varphi_{n\mathbf{k}}}$. The summation is performed over a set of values of the wave vector $\mathbf{k}$; the values are determined by the boundary conditions. For the sake of simplicity, we present the quantum Hamiltonian (5) for the two-mode ($\mathbf{k_1} \neq \mathbf{k_2}$) field $\mathbf{A}(\mathbf{r},t) = \mathbf{A}_{\mathbf{k}_1}(\mathbf{r},t) + \mathbf{A}_{\mathbf{k}_2}(\mathbf{r},t)$:

$$\hat{\mathcal{H}} = \hbar\omega_1 \left(\hat{\mathcal{N}}_{\mathbf{k_1}} + \frac{1}{2} \right) + \hbar\omega_2 \left(\hat{\mathcal{N}}_{\mathbf{k_2}} + \frac{1}{2} \right) + \tag{12}$$

$$+ \frac{\hbar(\omega_1\omega_2)^{1/2}}{2} \left(\frac{\hat{\mathbf{a}}^{\dagger}_{\mathbf{k_1}} \hat{\mathbf{a}}_{\mathbf{k_2}}}{V} \int e^{-i\mathbf{k}_1\mathbf{r} - i\varphi_1} e^{i\mathbf{k}_2\mathbf{r} + i\varphi_2} dV \right) + \tag{13}$$

$$+ \frac{\hbar(\omega_1\omega_2)^{1/2}}{2} \left(\frac{\hat{\mathbf{a}}_{\mathbf{k_1}} \hat{\mathbf{a}}^{\dagger}_{\mathbf{k_2}}}{V} \int e^{i\mathbf{k}_1\mathbf{r} + i\varphi_1} e^{-i\mathbf{k}_2\mathbf{r} - i\varphi_2} dV \right) + \tag{14}$$

$$+ \frac{\hbar(\omega_1\omega_2)^{1/2}}{2} \left(\frac{\hat{\mathbf{a}}^{\dagger}_{\mathbf{k_2}} \hat{\mathbf{a}}_{\mathbf{k_1}}}{V} \int e^{-i\mathbf{k}_2\mathbf{r} - i\varphi_2} e^{i\mathbf{k}_1\mathbf{r} + i\varphi_1} dV \right) + \tag{15}$$

$$+ \frac{\hbar(\omega_1\omega_2)^{1/2}}{2} \left(\frac{\hat{\mathbf{a}}_{\mathbf{k_2}} \hat{\mathbf{a}}^{\dagger}_{\mathbf{k_1}}}{V} \int e^{i\mathbf{k}_2\mathbf{r} + i\varphi_2} e^{-i\mathbf{k}_1\mathbf{r} - i\varphi_1} dV \right). \tag{16}$$

Here, we used the non-conventional commutation relations to take into account the interference correlation between the modes: $[\hat{\mathbf{a}}_{\mathbf{k}_1}, \hat{\mathbf{a}}^{\dagger}_{\mathbf{k}_2}]$=1 and $[\hat{\mathbf{a}}_{\mathbf{k}_1}, \hat{\mathbf{a}}_{\mathbf{k}_2}]$=$[\hat{\mathbf{a}}^{\dagger}_{\mathbf{k}_1}, \hat{\mathbf{a}}^{\dagger}_{\mathbf{k}_2}]$=0. The Hamiltonian describes the non-conservation of energy and momentum in a quantum electromagnetic field. The term (12) is the traditional Hamiltonian of a quantum multimode EM field [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The term is responsible for the interference of the modes with themselves. The terms (13-16) describe the mode-mode correlation interference phenomenon. A simple analysis shows that the terms are responsible for the positive and negative correlation energies and for the negative [4] and positive probabilities in the quantum field. Notice that $[\hat{\mathbf{a}}_{\mathbf{k}}, \hat{\mathbf{a}}^{\dagger}_{\mathbf{k}}] \approx 0$ for the big values of $\langle\mathcal{N}_{\mathbf{k}}\rangle$. In such a case, the energy calculated by the quantum Hamiltonians (6, 12-16) has the classical value (1). We have presented the formulas for the coherent classical and quantum waves. The equations (1-16) can be easily rewrited for the particular cases of non-coherent or partially coherent classical or quantum waves. Notice that according to the Hamiltonian (6, 12-16) the zero fluctuations in the field energy are obtained in the vacuum state of non-coherent waves.

Let us now demonstrate that the creation and destruction of energy in EM field do associate with several basic physical phenomena. We start with consideration of classical EM fields. Analysis of Eqs. (1-4) shows that the experimental realization of the phase conditions required for the creation or destruction of energy is practically impossible in conventional physical systems (see also the study [12] and references therein). In the study [11], however, we have showed that the waves generated by the point-like sources separated by the distance $\Lambda < \lambda$ (for example, subwavelength gratings) satisfy the phase conditions in the far-field diffraction zone. The creation and destruction of energy on the subwavelength scale are associated with the recently discovered extraordinary transmission of light through subwavelength apertures in metal screens [17]. Indeed, in such experiments the wave vectors of light waves produced by the subwavelength apertures are practically the same, $\mathbf{k}_n \approx \mathbf{k}$. The cross correlation term in Eq. (1) creates or destroys the field energy. Another example is the creation or destruction of energy in a classical light pulse (wavepacket) propagating in a dispersive medium in the same direction ($\mathbf{k}_n/k_n = \mathbf{k}/k$). The wave phases of the different Fourier $\mathbf{k}$-components of the wavepacket change under the propagation. According to Eqs. (1-4) the phase modification leads to variation of the pulse energy. The energy can be created or destroyed if the amount of energy spent on the phase modification is smaller than the energy of the Fourier components. The phenomenon is relevant to the recent investigations of the momentum of light in a material medium [16] and the energy of ultra-short light pulses scattered by subwavelength apertures [18]. Taking into account the cross correlation energy can be important for understanding the energy conservation in other wave processes (for example, see the studies [12, 13, 14, 15] and references therein). Notice that the cross correlation energy term vanishes if the light waves have the spatial dependence appropriate for a cavity resonator. The resonator modes are orthogonal to each other.

We now demonstrate that the creation and destruction of energy in quantum EM field do associate with several basic quantum phenomena. Let us consider the energy of two photons. It is generally accepted that the energy of two photons is always constant. A simple analysis of the Hamiltonian (12-16) shows that the energy of two coherent photons (correlated photons or a biphoton) with $\hbar\omega_1 = \hbar\omega_2 = \hbar\omega$ depends on the wave vectors and phases, $0 \leq \langle\mathcal{E}\rangle \leq 4\hbar\omega$. The energy of two non-correlated photons only is preserved, $\langle\mathcal{E}\rangle = 2\hbar\omega$. The fact that energy (wavelength $\lambda = 2\pi\hbar c/\langle\mathcal{E}\rangle$) of a biphoton depends on the experimental configuration of the biphoton generation, in our model on the wave vectors and phases of the correlated photons, is well known. In Ref. [11], we have predicted the breaking of energy conservation law in any subwavelength physical system by taking into account the interference properties of Young's double-source subwavelength system. It is important to consider the quantum interference properties using the Hamiltonian (12-16). In conventional Young's setup, the two plane waves generated by the two pinholes separated by the distance $\Lambda >> \lambda$ have different wave vectors, $\mathbf{k}_1 \neq \mathbf{k}_2$ [11]. According to the expressions (13-16), the cross correlation term vanishes. Respectively, the energy of photons is conserved. In the case of Young's subwavelength system: $\Lambda << \lambda$ and $\mathbf{k}_1 = \mathbf{k}_2$. Thus, the interference completely destroys the energy at the phase condition $\varphi_1 - \varphi_2 = \pi$. The system creates energy in the case of $\varphi_1 - \varphi_2 = 0$. It is interesting that according to the Hamiltonian (12-16) the interference of a single photon with itselve does not create or destroy energy. In such a case, one of the plane waves characterized by the wave vector $\mathbf{k}_1$ or $\mathbf{k}_2$ does not contain the photon. In the case of diffraction of two coherent (correlated) photons, the biphoton energy ($0 \leq \langle\mathcal{E}\rangle \leq 4\hbar\omega$) depends on the wave phases φ_1 and φ_2. Finally, a simple analysis of the Hamiltonian (12-16) shows that a quantum entangled state of photons is preserved on passage through an aperture array. The propagation of a correlated (entangled) state through an optically thick lens, however, destroys the correlation by the well-known modification of the wave vectors $\mathbf{k}$ and phases $\varphi_{\mathbf{k}}$. Such a behavior is in agreement with the recent experiment [20].

We now present examples of the creation and destruction of energy in other basic quantum systems. Let us consider a Dicke superradiance quantum model [19] of emission of a subwavelength ensemble of atoms. In the model, the wave vectors of the light waves produced by the atoms in the far-field zone are practically the same, $\mathbf{k}_n \approx \mathbf{k}$. According to Ref. [19] and the Hamiltonian (12-16) the energy scales like $\langle\mathcal{E}\rangle = N^2\hbar\omega$, where N is the number of the correlated atoms (photons). In addition, our model predicts the destruction of energy in the Dicke ensemble of atoms at the condition $\varphi_1 - \varphi_2 = \pi$ for the atom (photon) pairs. In the present paper, we investigated the photon fields. Nevertheless, one can easily demonstrate the creation and destruction of energy in an ensemble of material particles. Let us consider the Bose-Einstein condensation in a Bose gas using the well-known equivalence between a Bose gas of N particles and a set of N quantized harmonic oscillators [10]. The Bose-Einstein condensation of the non-correlated particles leads to the phase correlation between them. According to the aforementioned equivalence and the Hamiltonian (12-16), the condensation leads to the phase transition from the non-correlated state having the energy $\langle\mathcal{E}\rangle = N\langle\mathcal{E}\rangle_1$ to the correlated state with the energy $\langle\mathcal{E}\rangle = N^2\langle\mathcal{E}\rangle_1$. The phase transition is accompanied with the increase of the system energy leading to a sudden explosion of the system like that in the so-called "bosenova" process. At the phase condition $\varphi_1 - \varphi_2 = \pi$ for the boson pairs the original atoms in the condensate have to be vanished [21]. At the same phase conditions our model predicts the gas superfluidity associated with the condensation [22, 23], $\langle\mathcal{E}\rangle = 0$. For a Fermi electron gas, the phase transition can lead to the superconductivity of the gas. Indeed, the non-correlated electron state having the energy $\langle\mathcal{E}\rangle = \langle\mathcal{E}\rangle_{Fermi}$ can transit to the correlated state with the energy $\langle\mathcal{E}\rangle = 0$ at the appropriate phase conditions. The Fermion field analog of the Hamiltonian (12-16), which will be presented in our next paper, shows that the electrons in the Cooper pairs should satisfy the phase condition $\varphi_1 - \varphi_2 = \pi$.

In conclusion, we presented a quantum model of the creation and destruction of energy in an electromagnetic field. The Hamiltonian describing the non-conservation of energy was derived. We have showed that the energy non-conservation is associated with many basic physical phenomena, such as the extraordinary transmission of light through subwavelength apertures, scattering of entangled photons in Young's two-slit experiment, Dicke's quantum superradiance, Bose-Einstein condensation, bosenova effect, zero energy of Cooper's electron pairs, and Feynman's negative probabilities in quantum fields.

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